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TrinLat Collaboration

Lattice Quark Propagator (I)

Compute All Elements of the

Improving Algorithms to

Improving Algorithms to Compute All Elements of the Lattice Quark Propagator (I)

Outline

- Summary of Part I
- the stochastic method
- combine exact low eigenmodes and
- Hybrid Method
- Noise-Dilution*
- Stochastic Estimation
- Spectral Decomposition
- Exact All-to-all Propagator
- Introduction

- Better operators and the variational method
 - disconnected diagrams
 - point propagators not sufficient
 - Flavour singlet physics
 - point propagators would be a huge waste
 - from the expensive configurations
 - want to get as much information as one can
 - configurations in the near future
 - Anticipating small number of very expensive full QCD
- The need for all-to-all propagators**

Introduction

- noisy method (but in the least noisy way)
- solve for low eigenmodes and correct for the truncation using the

What's the solution then?

- * exact but **truncation**
- get a finite number of the low lying modes exactly
- Spectral Decomposition
- average over random sources \rightarrow noisy
- Stochastic Estimates
- typically more than a million quark inversions
- $N^x * N^y * N^z * N^t * N^{spin} * N^{colour}$ inversions

What's the problem then?

brutal truncation

$$\text{Truncated propagator} = \sum_{N^{\epsilon_a}} \frac{1}{\lambda^i} v^{(i)}(\underline{x}, t) \otimes v^{(i)\dagger}(\underline{x}_0, t_0) \gamma^5$$

$$(v^{(i)})^\dagger = \mathcal{O} v^{(i)}$$

Solve low-lying eigenvectors, $v^{(i)}(\underline{x}, t)$, and their eigenvalues, λ^i

$$\text{Hermitian Dirac Matrix} \quad \mathcal{O} = \gamma^5 M$$

much of the important physics (Barddeen *et al.*, SESAM, ...)

- A small number of the low lying modes solved exactly will capture

The physics in the low lying modes

Spectral Decomposition

(various methods of variance reduction C.Michael *et al.*, ...)

\leftrightarrow but is noisy

of random noise sources

Unbiased estimate of the all-to-all quark propagator with N^{st} samples

- Quark propagator = $\langle \psi_{(A)}(\underline{x}, t) \otimes \eta_{(A)\dagger}(\underline{x}_0, t_0) \rangle$

- solution $\psi_{(A)}(\underline{x}, t) = M^{-1}_{-1}(\underline{x}, t; \underline{x}_0, t_0) \eta_{(A)}(\underline{x}_0, t_0)$

with $\langle \eta_{(A)} \eta_{(B)\dagger} \rangle = \delta_{AB}$

- create noise source $\eta_{(A)}(\underline{x})$

Average over many random samples on each configuration,

Stochastic Estimation

$$\sum_{N^d} \phi_i(x_0, t_0) \otimes \phi_a^m(x, t) = \phi_I(x, t; x_0, t_0)$$

where $\phi_i = \eta^{(i)}$. The all-to-all quark propagator is then,

$$\phi = \eta^{(1)} + \eta^{(2)} + \eta^{(3)} + \dots + \eta^{(N^d)}$$

where the vectors $\eta^{(i)}$'s are mostly zero. Solution is,

$$\eta = \eta^{(1)} + \eta^{(2)} + \eta^{(3)} + \dots + \eta^{(N^d)}$$

Dilute the random noise vector, η

Diluting the Noise

$$\eta_{(3)} + \eta_{(2)} + \eta_{(1)}$$

$$\begin{vmatrix} \eta_0^3 & 0 & 0 \\ 0 & \eta_0^2 & 0 \\ 0 & 0 & \eta_0^1 \\ 0 & 0 & 0 \end{vmatrix} + \begin{vmatrix} 0 & \eta_0^3 & 0 \\ 0 & \eta_0^2 & 0 \\ 0 & 0 & \eta_0^1 \\ 0 & 0 & 0 \end{vmatrix} + \begin{vmatrix} 0 & 0 & \eta_0^3 \\ 0 & 0 & \eta_0^2 \\ 0 & 0 & \eta_0^1 \\ 0 & 0 & 0 \end{vmatrix}$$

$$\leftarrow \begin{vmatrix} \eta_0^3 & \eta_0^3 & \eta_0^3 \\ \eta_0^2 & \eta_0^2 & \eta_0^2 \\ \eta_0^1 & \eta_0^1 & \eta_0^1 \\ \eta_0^0 & \eta_0^0 & \eta_0^0 \end{vmatrix} = (\vec{x}, t) \eta_c^s$$

Color Dilution

* full dilution \equiv full all-to-all propagator!

Continue diluting . . .

- Spin dilution, Even-Odd (space) dilution, etc. etc.

$\eta^{(i)}$ have only nonzero entries for colour index $a = i$

- Colour dilution $N_{dil}^i = N_{colour}^i$

... like wall source on every timeslice (Fukugita et al.)

$\eta^{(i)}$ have only nonzero entries on timeslice $t = i$

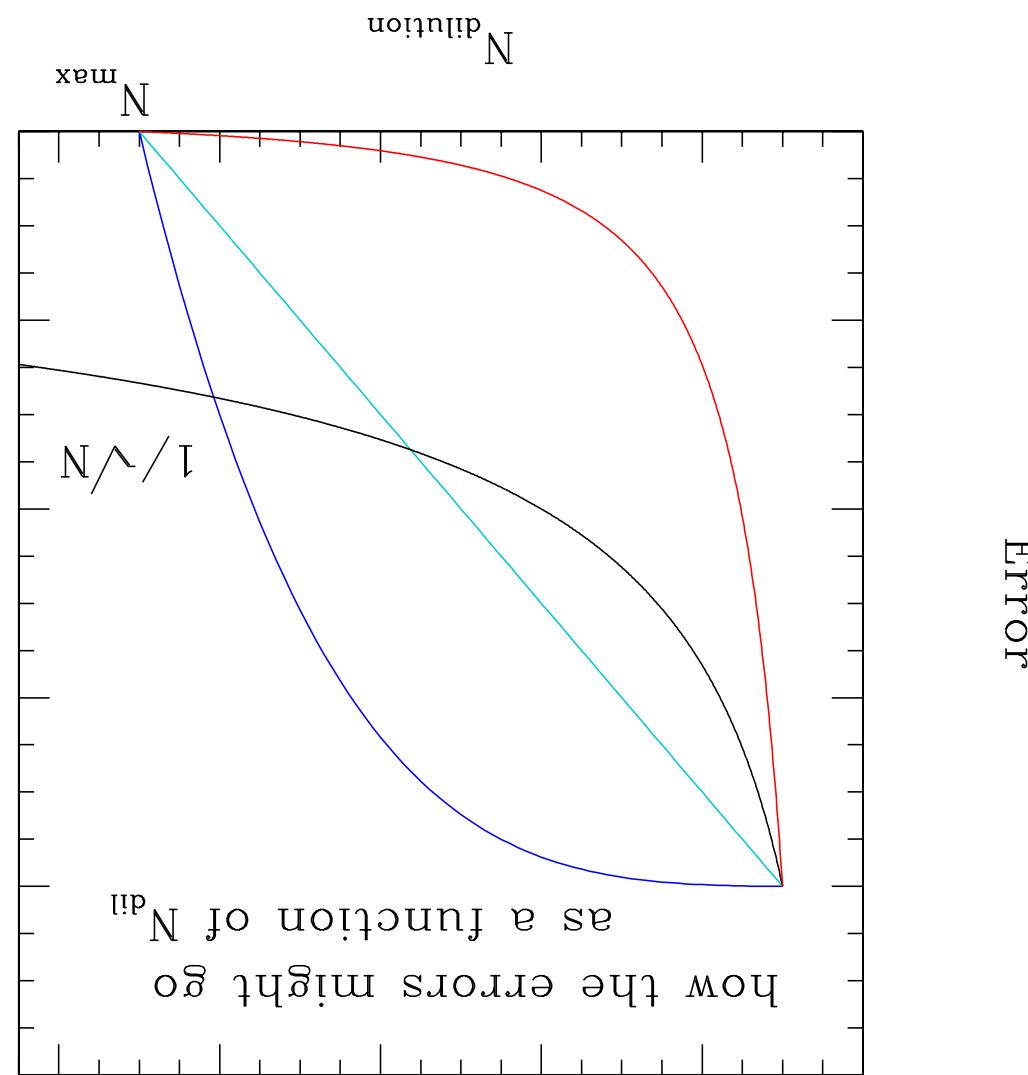
- Time dilution $N_{dil}^t = N^t$

Examples of Dilutions

Improving Algorithms to Compute All Elements of the Lattice Quark Propagator (I)

- If you choose the dilution incorrectly, then dilution will not decrease the errors eg. diluting two components that do not communicate with each other
- But if chosen wisely, one get a large gain variance from noise vectors can be effectively reduced to zero before reaching the "homeopathic limit"
(where the noise from the gauge configurations dominate)
- the best dilution may depend on the application of interest (some tuning involved)
(Wilcox, SESAM)

- exploit $\eta = \eta_{(1)} + \eta_{(2)}$ and $\phi = \phi_{(1)} + \phi_{(2)}$ for further dilutions



$$\left\{ (0_t)_{(i)}^{[0]} \not{\partial}_t \gamma_5 \Gamma_\dagger^{[1]} u_{\dagger(i)}^{[0]} \right\} \sum_{N^p}^{i=1} \times \\ \left\{ (t\nabla + 0_t)_{(j)}^{[1]} \not{\partial}_t \gamma_5 \Gamma_\dagger^{[1]} u_{\dagger(j)}^{[1]} \right\} \sum_{N^p}^{j=1} \sum_{t_0}^{j=1} = C_{disc}(\Delta t)$$

Disconnected Correlator

where i, j are dilution indices.

$$\left[(0_t)_{(i)}^{[0]} \not{\partial}_t (0_t)_{(j)}^{[1]} u \right] \left[(t)_{(j)}^{[1]} \not{\partial}_t (t)_{\dagger(i)}^{[0]} u_\dagger \right] \sum_{N^p}^{i=1} \sum_{t_0}^{j=1} \sum_{j=1}^{i=1} = \\ < (0_t)^{[0]} \underline{\not{\partial}}^{[1]} \gamma_5 \underline{\Gamma}^{[1]} (t)^{[0]} \gamma_5 \underline{\Gamma}^{[1]} (t_0) > = C_u(\Delta t)$$

Pseudoscalar Correlator

$$\begin{aligned}
 & \mathcal{O}^0 + \mathcal{O}^1 = \\
 & \sum_{N^{ev}}^{N^{ev}+1} \chi^i_{\langle i} \otimes \chi^i_{\rangle i} + \sum_{N^{ev}}^i \chi^i_{\langle i} \otimes \chi^i_{\rangle i} = \mathcal{O} \\
 V &= V^0 \oplus V^1
 \end{aligned}$$

subspaces, V^0 and V^1 .

The truncation naturally divides the space of solutions, V , into two

- Want to do this without losing the low modes
- Correct for the truncation with the "noisy" method
- Solved for N^{ev} lowest eigenmodes exactly

Correcting the Truncation

modes.

can correct for the truncation without introducing noise in the low

By projecting the "noisy" sources onto V_1 with $P_{1\eta} = (1 - P_0)\eta$, we

$$\begin{aligned} P_2^0 &= P_0 \\ P_2^1 &= P_1 \\ P_0 + P_1 &= 1 \quad P_0 P_1 = 0 \end{aligned}$$

$$P_0 = \sum_{N_{e^a}}^{i=1} u_{(i)}^\dagger \otimes u_{(i)}$$

Define the projection operators,

... So we just need to get \underline{O}_1

we have $\underline{O}_{-1} = \underline{O}_0 + \underline{O}_1$.

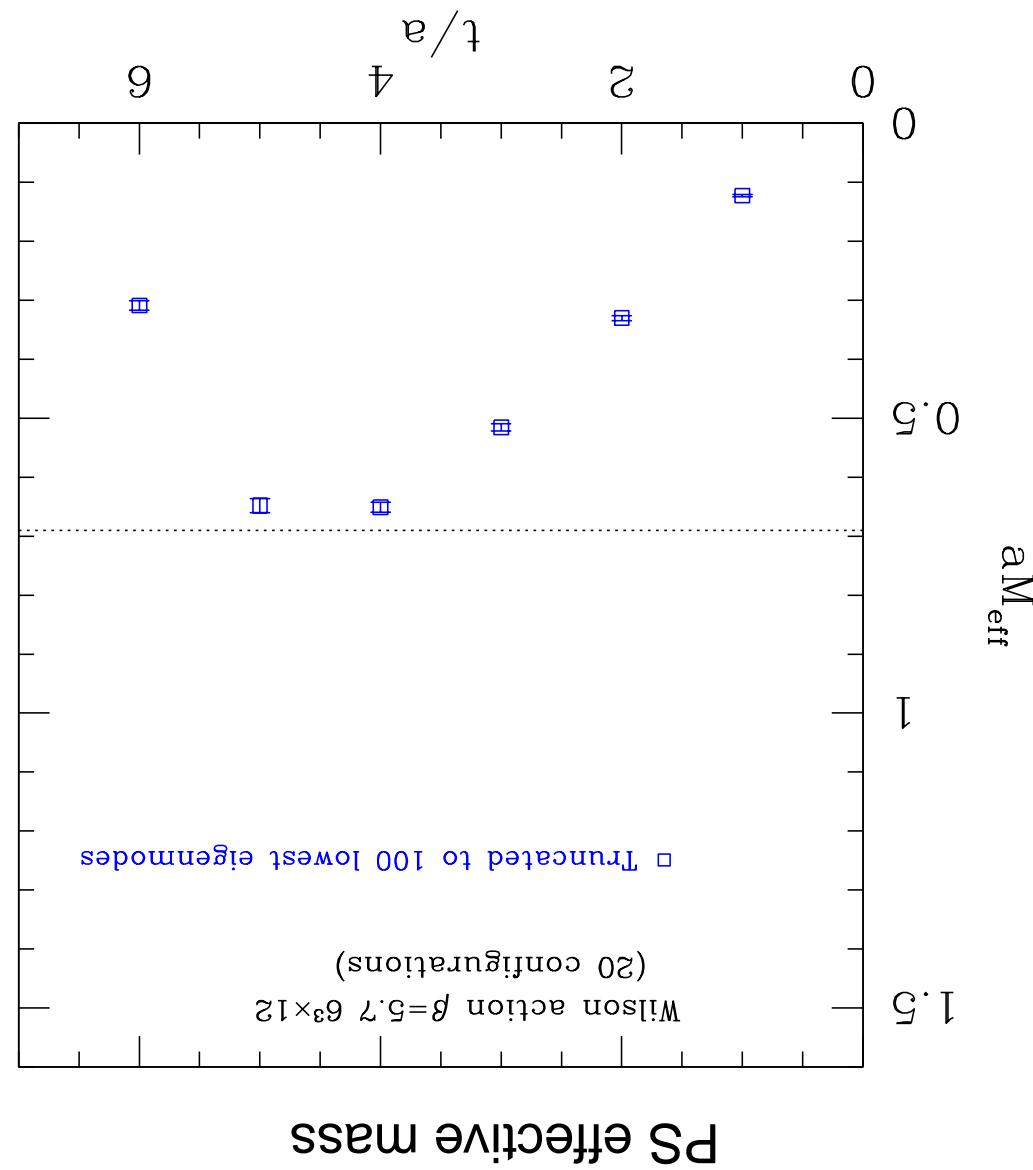
Defining $\underline{O}_0 \equiv \sum_{N_{e^a}} \frac{1}{N_{e^a}} u_i^\dagger \otimes u_{(i)^\dagger}$ and $\underline{O}_1 \equiv \sum_{N_{e^a}+1} \frac{1}{N_{e^a}+1} u_i^\dagger \otimes u_{(i)^\dagger}$

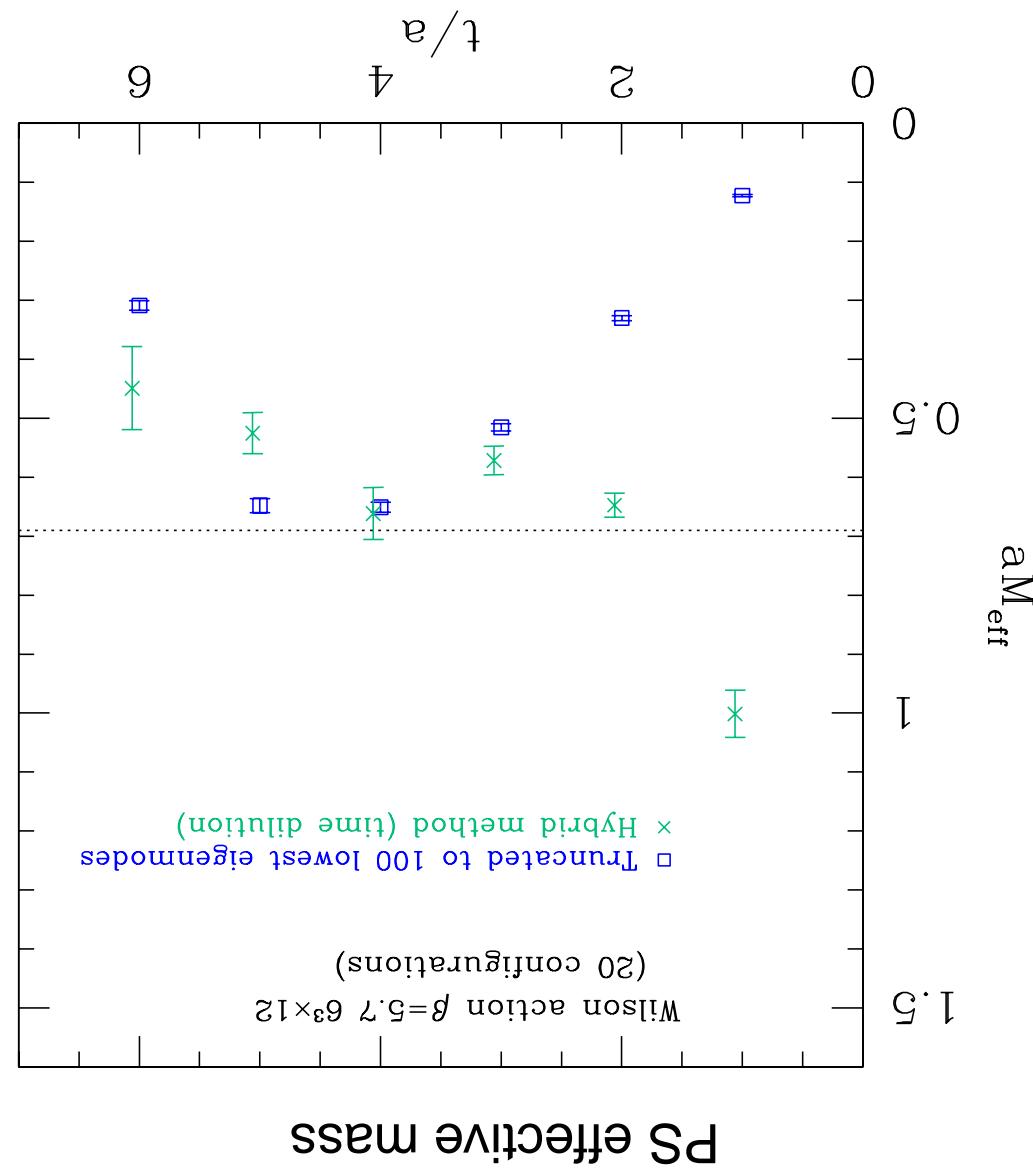
$$\begin{aligned} u(0) - 1 &= \\ (P^1 u) &= \phi O \end{aligned}$$

where $\langle \langle u_i \otimes u_j \rangle \rangle$ and ϕ is the solution to,

$$\begin{aligned} \langle \langle u_i \otimes \phi \rangle \rangle + {}^0\bar{O} &= \\ \langle \langle u_i \otimes u_j \rangle \rangle P_{-1} O + {}^0\bar{O} &= \\ P_{-1} O + {}^0\bar{O} &= \\ O_{-1} + {}^0\bar{O} &= \end{aligned}$$

In other words,





$$u^{(i)} = \left\{ v_{(1)}, v_{(2)}, \dots, v_{N^{ea}}, u_{(1)}, u_{(2)}, \dots, u_{N^{di}} \right\}$$

$$u^{(i)} = \left\{ \frac{1}{N^{ea}} \sum_{j=1}^{N^{ea}} v_{(1)}, v_{(2)}, \dots, v_{(N^{ea})}, \frac{1}{N^{di}} \sum_{j=1}^{N^{di}} u_{(1)}, u_{(2)}, \dots, u_{(N^{di})} \right\}$$

("hybrid") list,

One can naturally combine the two approaches by forming a long

$$G = \sum_{N^a} u^{(i)} \otimes v_{(i)}(x^0, t^0)$$

- Noise Method

$$G = \sum_{N^a} v_{(i)} \otimes u^{(i)}(x^0, t^0)$$

- Spectral Decomposition

Hybrid List Method

and colour indices.

where $[0]$ and $[1]$ are flavour indices and $\{ \}$ indicate a trace over spin

$$C_{discoun} = \sum_{N^{list}}^i \left\{ (t_0)_{(i)}^{[0]} u_{\dagger(i)}^{[0]} (t_0) \gamma^5 T_{\dagger} u_{(j)}^{[1]} (t) \right\} \sum_{N^{list}}^j \left\{ (t)_{(j)}^{[1]} u_{\dagger(j)}^{[1]} (t) \gamma^5 T_{\dagger} u_{(i)}^{[0]} (t_0) \right\}$$

Disconnected pieces,

$$C_{AB} = \sum_{N^{list}}^i \sum_{N^{list}}^j \left\{ (t)_{(i)}^{[0]} u_{\dagger(i)}^{[0]} (t) \gamma^5 T_{\dagger} u_{(j)(A)}^{[1]} (t_0) \gamma^5 T_{\dagger} u_{(j)(B)}^{[0]} (t_0) \right\}$$

Meson Correlation Functions

$$G = \sum_{N^{list}}^i u_{(i)}(\underline{x}, t) \otimes u_{\dagger(i)}(\underline{x}_0, t_0) \gamma^5$$

The all-to-all quark propagator is then simply,

- stochastic method can exactly correct for the truncation without ruining the exactly solved low eigenmodes
- depending on the problem, one can increase/decrease the number of the exactly solved modes (**trunable**)
- operator construction becomes much easier and more intuitive ($\psi^\dagger T \psi$ type construction)
- dilution will be needed to keep the noise level down recall that dilution method gives the exact all-to-all in a finite number of steps anyway

- 1 Determine some number of low lying eigenvalues and eigenmodes
- 2 Estimate the best lowest level of dilution
- 3 Solve for all of the N^{dil} solutions $\{\psi^{(d)}\}$
- 3 Construct the hybrid list and contract all indices

Recipe

- one cannot lose (except for disk space!)
- # of exact modes and # of dilutions can be tuned
- **Hybrid List** allows a natural way to combine the methods
(expect small variance if dilution is chosen appropriately)
- **Dilution** method has zero variance in the homeopathic limit
- correct for the truncation with stochastic method
- solve "a few" exactly
- for very light quarks, **exact low eigenmodes** are important
- No more "point quark propagators"!

Summary of Part I

Does it? next talk

- * **Dilution** must keep the noise level down for this to work